TA Session, Problem Set 1 and Matlab Camera Calibration

CS231a

January, 17, 2014

Announcements

- PS1 out, due January 28th, start of class
  - Material coverage through 1/21 Lecture
  - Some Matlab programming required
  - Reading HZ Ch 8 might be helpful.
- Project discussion section next Friday
- Matlab Review session Wednesday 2:15-3:05, Nvidia Auditorium

Lagrangian Multipliers

Question 3 asks you to minimize an expression subject to some constraints. Written as:

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & g_1(x) = 0 \\ & g_2(x) = 0 \\ & \dots \end{aligned}$$

where $f : \mathbf{R}^n \to \mathbf{R}$ and $g_i : \mathbf{R}^n \to \mathbf{R}^p$
Adapted from EE263 Notes

Without constraints, minimizing $f(x)$ where $f : \mathbf{R}^n \to \mathbf{R}$ means selecting an optimal x that satisfies:

$$\bigtriangledown f(x) = 0$$

So we can find all such points that satisfy $\bigtriangledown f(x) = 0$ and the minimum must be one of them.

However adding constraints complicates this, but we can compensate by forming the *Lagrangian*

$$L(x, \lambda) = f(x) + \lambda^T g(x)$$

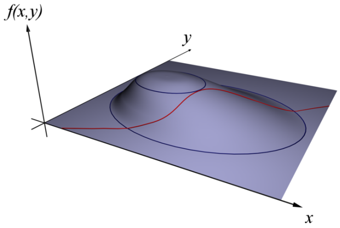
where λ^T is a $\mathbf{R}^p$ vector called a Lagrange multiplier.

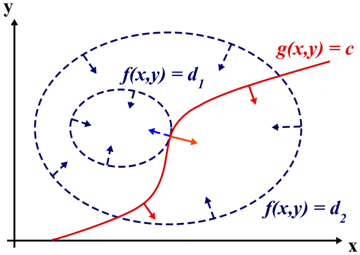
To find the minimum of $f(x)$ with the constraint $g(x)$, we leverage the two optimality conditions

$$\bigtriangledown_x L(x, \lambda) = 0, \quad \bigtriangledown_\lambda L(x, \lambda) = g(x) = 0$$

Notice that the left most quantity is just the original constraint.

The geometric intuition behind this can be seen below:





Thus if we want a point where the gradients of f and g are parallel and the constraint $g(x) = c$ holds

$$\bigtriangledown_x f = -\lambda \bigtriangledown_x g$$

$$and$$

$$g(x) = c$$

These two expression can be combined into the Lagrangian shown earlier

$$L(x, \lambda) = f(x) + \lambda^T g(x)$$

and solved by evaluating

$$\bigtriangledown_{x,\lambda} L(x, \lambda) = 0$$

Matrix Derivatives

Setting up the Lagrangian is only one part of Problem 3, to solve the minimization problem we must evaluate the gradient of the Lagrangian by taking the partial derivative with respect to x_i's and λ_i's.

Complications:

- Your expressions will involve matrices
- The expression that you will minimize takes vectors as inputs
- You will have to take partial derivatives with respect to a vector

Possible useful identities

Scaler by vector derivatives are defied as

$$\frac{\partial y}{\partial \mathbf{x}} = \left[\frac{\partial y}{\partial x_1} \frac{\partial y}{\partial x_2} \dots \frac{\partial y}{\partial x_n} \right]^T$$

where y is a scaler and $\mathbf{x} = [x_1 x_2 \dots x_n]^T$

The following identities may be helpful.

$$\frac{\partial x^T A x}{\partial x} = \left(A + A^T \right) x$$

$$\frac{\partial b^T A x}{\partial x} = A^T b$$

$$\frac{\partial x^T x}{\partial x} = 2x$$

Matrix Lagrangian Example

Linearly constrained least squares

$$\begin{array}{rl} \text{minimize} & (1/2) \parallel Ax - b \parallel^2 \\ \text{subject to} & Cx - d = 0 \end{array}$$

With $A \in \mathbf{R}^{m \times n}$ and $C \in \mathbf{R}^{p \times n}$

What is the Lagrangian and corresponding optimality conditions?

Demo

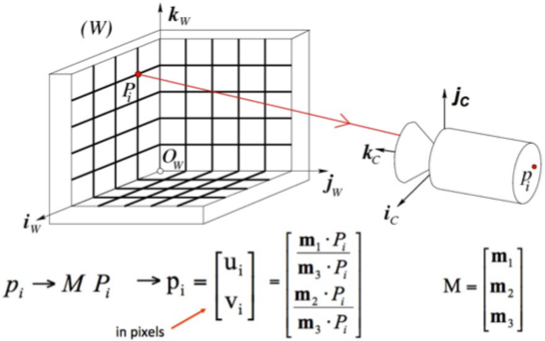


Figure : Lecture camera calibration example

`http://www.vision.caltech.edu/bouguetj/calib_doc/index.html`