Lecture 4

Single View Metrology

• Review calibration
• Vanishing points and lines
• Estimating geometry from a single image
• Extensions

Reading:
[HZ] Chapter 2 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 3 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 8 “More Single View Geometry”
[Hoeim & Savarese] Chapter 2
Calibration Problem

\[ \text{World ref. system} \rightarrow \text{In pixels} \]

\[ P_i \rightarrow M \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \]

\[ M = K [R \quad T] \]

\[ K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o \\ 0 & \beta \sin \theta & v_o \\ 0 & 0 & 1 \end{bmatrix} \]
Calibration Problem

\[ M = K[R \quad T] \]

11 unknown
Need at least 6 correspondences

World ref. system  In pixels
Once the camera is calibrated...

\[ M = K[R \quad T] \]

- Internal parameters $K$ are known
- $R$, $T$ are known – but these can only relate $C$ to the calibration rig

Can I estimate $P$ from the measurement $p$ from a single image?

No - in general $\ominus [P$ can be anywhere along the line defined by $C$ and $p]$
Recovering structure from a single view

unknown

known

Known/
Partially known/
unknown
Recovering structure from a single view

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl
Transformation in 2D

- Isometries
- Similarities
- Affinity
- Projective
Transformation in 2D

Isometries:

\[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix}
= \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
= H_e
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

- Preserve distance (areas)
- 3 DOF
- Regulate motion of rigid object
Transformation in 2D

Similarities:

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix}
= \begin{bmatrix}
  s & R & t \\
  0 & 1 & 0 \\
  1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
= H_s
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

- Preserve
  - ratio of lengths
  - angles
- 4 DOF
Transformation in 2D

Affinities:

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  A & t \\
  0 & 1
\end{bmatrix}\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix} = H_a \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

\[A = \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi)\]

\[D = \begin{bmatrix}
  s_x & 0 \\
  0 & s_y
\end{bmatrix}\]
Transformation in 2D

Affinities:

\[
\begin{bmatrix}
    x' \\
    y' \\
    1
\end{bmatrix}
= \begin{bmatrix}
    A & t \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
= H_a
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
\]

\[A = \begin{bmatrix}
    a_{11} & a_{12} \\
    a_{21} & a_{22}
\end{bmatrix}
= R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi)
\]

\[D = \begin{bmatrix}
    s_x & 0 \\
    0 & s_y
\end{bmatrix}
\]

-Preserve:
- Parallel lines
- Ratio of areas
- Ratio of lengths on collinear lines
- others...
- 6 DOF
Transformation in 2D

Projective: 
\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = 
\begin{bmatrix}
  A & t \\
  v & b
\end{bmatrix} 
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix} = H_p 
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

- 8 DOF
- Preserve:
  - cross ratio of 4 collinear points
  - collinearity
  - and a few others...
The cross ratio

The cross-ratio of 4 collinear points

\[
\begin{vmatrix}
\|P_3 - P_1\| & \|P_4 - P_2\| \\
\|P_3 - P_2\| & \|P_4 - P_1\|
\end{vmatrix}
\]

Can permute the point ordering

\[
\begin{vmatrix}
\|P_1 - P_3\| & \|P_4 - P_2\| \\
\|P_1 - P_2\| & \|P_4 - P_3\|
\end{vmatrix}
\]

\[
P_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}
\]
Lines in a 2D plane

ax + by + c = 0

\[
\begin{bmatrix}
    a \\
    b \\
    c
\end{bmatrix}
\]

\[
l = \begin{bmatrix}
    a \\
    b \\
    c
\end{bmatrix}
\]

If \( x = [x_1, x_2]^T \in l \)

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    1
\end{bmatrix}^T \begin{bmatrix}
    a \\
    b \\
    c
\end{bmatrix} = 0
\]
Lines in a 2D plane

Intersecting lines

\[ x = l \times l' \]

Proof

\[ l \times l' \perp l \] \quad \rightarrow \quad (l \times l') \cdot l = 0 \quad \rightarrow \quad x \in l
\]

\[ l \times l' \perp l' \] \quad \rightarrow \quad (l \times l') \cdot l' = 0 \quad \rightarrow \quad x \in l'

\[ x \]

\[ \rightarrow x \text{ is the intersecting point} \]
2D Points at infinity (ideal points)

\[
x = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}, \quad x_3 \neq 0
\]

\[
x_\infty = \begin{bmatrix}
x'_1 \\
x'_2 \\
0
\end{bmatrix}
\]

Let’s intersect two parallel lines:

\[
\rightarrow l \times l' \propto \begin{bmatrix}
b \\
-a \\
0
\end{bmatrix} = x_\infty
\]

- In Euclidian coordinates this point is at infinity
- Agree with the general idea of two lines intersecting at infinity
2D Points at infinity (ideal points)

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad x_3 \neq 0 \]

Note: the line \( l = [a \ b \ c]^T \) pass trough the ideal point \( x_{\infty} \)

\[ l^T x_{\infty} = [a \ b \ c] \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = 0 \]

So does the line \( l' \) since \( a \ b' = a' \ b \)
Lines infinity $1_{\infty}$

Set of ideal points lies on a line called the line at infinity

How does it look like?

$$1_{\infty} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Indeed:

$$\begin{bmatrix} x_1^T \\ x_2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

A line at infinity can thought of the set of “directions” of lines in the plane
Projective transformation of a point at infinity

\[ H = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \]

\[ p' = H \cdot p \]

\[ H \cdot p_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix} \]

...no!

An affine transformation of a point at infinity is still a point at infinity
Projective transformation of a line (in 2D)

\[ H = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \]

\[ l' = H^{-T} l \]

is it a line at infinity?

\[ H^{-T} l_\infty = ? = \begin{bmatrix} A & t \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ b \end{bmatrix} \] ...no!

\[ H_A^{-T} l_\infty = ? = \begin{bmatrix} A \\ 0 \\ 0 \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A^{-T} \\ -t^T A^{-T} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
Points and planes in 3D

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \]

\[ x \in \Pi \iff x^T \Pi = 0 \quad ax + by + cz + d = 0 \]

How about lines in 3D?
- Lines have 4 degrees of freedom - hard to represent in 3D-space
- Can be defined as intersection of 2 planes
Vanishing points

In 3D, vanishing points are the equivalent of ideal points in 2D

Points where parallel lines intersect in 3D

\[ x_{\infty} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix} \]

\[ H_p \]

\[ p_{\infty} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \]
Vanishing points

In 3D, vanishing points are the equivalent of ideal points in 2D

Points where parallel lines intersect in 3D

\[ x_\infty = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix} = \text{direction of the line in 3D} \]
The horizon line

\[ l_{\text{hor}} = H_P^{-T} l_{\infty} \]
The horizon line
Planes at infinity & vanishing lines

- Parallel planes intersect the plane at infinity in a common line – the vanishing line (→ horizon)
- A set of vanishing lines defines the plane at infinity $\Pi_\infty$
- 2 planes are parallel iff their intersections is a line that belongs to $\Pi_\infty$

$\Pi_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

plane at infinity
Vanishing points and their image

d = direction of the line

\[ \mathbf{v} = \mathbf{K} \mathbf{d} \]

\[ \mathbf{x}_\infty = \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} \xrightarrow{M} \mathbf{v} = \mathbf{X}_\infty \mathbf{M} = \mathbf{K} \begin{bmatrix} 1 & 0 \\ a & b \\ 0 & c \end{bmatrix} = \mathbf{K} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]
Vanishing points - example

v1, v2: measurements
K = known and constant

Can I compute R?
No rotation around z

\[ \mathbf{d}_1 = \frac{K^{-1} \mathbf{v}_1}{\|K^{-1} \mathbf{v}_1\|} \]

\[ \mathbf{d}_2 = \frac{K^{-1} \mathbf{v}_2}{\|K^{-1} \mathbf{v}_2\|} \]

\[ \mathbf{R} \mathbf{d}_1 = \mathbf{d}_2 \rightarrow \mathbf{R} \]

In 2D

\[ \theta_R = \alpha - \beta \]
\[ d_1 = \frac{K^{-1}v_1}{\|K^{-1}v_1\|} \]
\[ d_2 = \frac{K^{-1}v_2}{\|K^{-1}v_2\|} \]
Vanishing lines and their images

Parallel planes intersect the plane at infinity in a common line – the vanishing line (horizon)

\[ \mathbf{n} = \mathbf{K}^T \mathbf{l}_{\text{horiz}} \]
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Estimating geometry & calibrating the camera from a single image

Are these two lines parallel or not?

- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- if yes, these 2 lines are // in 3D

• Recognition helps reconstruction!
• Humans have learnt this
Angle between 2 vanishing points

\[
\cos \theta = \frac{v_1^T \omega v_2}{\sqrt{v_1^T \omega v_1} \sqrt{v_2^T \omega v_2}} \quad \omega = (K K^T)^{-1}
\]

If \( \theta = 90 \) \( v_1^T \omega v_2 = 0 \)
Projective transformation of $\Omega_\infty$

$$\omega = P^{-T} \Omega_\infty P^{-1} = (K K^T)^{-1}$$

$$P = K \begin{bmatrix} R & T \end{bmatrix}$$

1. It is not function of $R, T$

2. $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$ symmetric

3. $\omega_2 = 0$ zero-skew

4. $\omega_1 = \omega_3$ square pixel

Absolute conic
Angle between 2 scene lines

\[ \theta = 90 \]

\[
\begin{cases}
    v_1^T \omega v_2 = 0 \\
    \omega = (K K^T)^{-1}
\end{cases}
\]

Constraint on K
Single view calibration - example

\[ \mathbf{v}_1^T \mathbf{\omega} \mathbf{v}_2 = 0 \quad \mathbf{\omega}_2 = 0 \]
\[ \mathbf{v}_1^T \mathbf{\omega} \mathbf{v}_3 = 0 \quad \mathbf{\omega}_1 = \mathbf{\omega}_3 \]
\[ \mathbf{v}_2^T \mathbf{\omega} \mathbf{v}_3 = 0 \]

\[ \mathbf{\omega} = \begin{bmatrix} \mathbf{\omega}_1 & \mathbf{\omega}_2 & \mathbf{\omega}_4 \\ \mathbf{\omega}_2 & \mathbf{\omega}_3 & \mathbf{\omega}_5 \\ \mathbf{\omega}_4 & \mathbf{\omega}_5 & \mathbf{\omega}_6 \end{bmatrix} \]

\[ \rightarrow \text{Compute } \mathbf{\omega} : \]

Once \( \mathbf{\omega} \) is calculated, we get \( \mathbf{K} \):

\[ \mathbf{\omega} = (\mathbf{K} \mathbf{K}^T)^{-1} \rightarrow \mathbf{K} \]

(Cholesky factorization; HZ pag 582)
Single view reconstruction - example

\[ n = K^T l_{\text{horiz}} \]

\( K \) known \( \rightarrow \) 

= Scene plane orientation in the camera reference system

Select orientation discontinuities
Single view reconstruction - example

Recover the structure within the camera reference system

Notice: the actual scale of the scene is NOT recovered

- Recognition helps reconstruction!
- Humans have learnt this

Are these two lines parallel or not?
- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- if yes, these 2 lines are \parallel in 3D
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[Hoeim & Savarese] Chapter 2
La Trinita' (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)
La Trinita' (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)
Manually select:
• Vanishing points and lines;
• Planar surfaces;
• Occluding boundaries;
• Etc..
Automatic Photo Pop-up

Hoiem et al, 05…
Automatic Photo Pop-up

Hoiem et al, 05…

Software:

Make3D

Training

Planar Surface Segmentation

Image

Depth

Prediction

Plane Parameter MRF

$P(\alpha|X, \nu, y, R; \theta) = \frac{1}{Z} \prod_i f_1(\alpha_i|X_i, \nu_i, R_i; \theta) \prod_{i,j} f_2(\alpha_i, \alpha_j|y_{ij}, R_i, R_j)$

Connectivity

Co-Planarity
Single Image Depth Reconstruction

Saxena, Sun, Ng, 05…

A software: Make3D
“Convert your image into 3d model”  http://make3d.stanford.edu/

http://make3d.stanford.edu/images/view3D/185
http://make3d.stanford.edu/images/view3D/931?noforward=true
Coherent object detection and scene layout estimation from a single image

Y. Bao, M. Sun, S. Savarese, CVPR 2010,
BMVC 2010
Next lecture:

Multi-view geometry (epipolar geometry)