CS 231A Computer Vision (Winter 2015)
Problem Set 0
Due Date: Jan 14$^{\text{th}}$ 2015 5:00pm

This is a short tutorial on how to use MATLAB and a review of some small linear algebra ideas. The point of this assignment is to get you used to manipulating matrices and images in MATLAB.

Make sure you use a ".m" file to write your matlab script and use the publish functionality in MATLAB to print your code and results. Please suppress all unnecessary results with semicolons.

This problem set is not representative of future problem sets in terms of length or difficulty, but the logistics will be similar (submission of code, Scoryst, Piazza, etc).

Submit your published results to the class Scoryst. Feel free to ask any questions to the class Piazza forum (but use a private post if you have code or specifics to discuss).

1 Piazza Poll (10 points)

Please register on the class Piazza forum and answer the poll about your class background.

2 Basic Matrix/Vector Manipulation (10 points)

In MATLAB, please calculate the following. Given matrix M and vectors a,b,c such that

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 2 & 2 \end{bmatrix}, \; a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \; b = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}, \; c = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

(a) Define Matrix M and Vectors a,b,c in Matlab

(b) Find the dot product of vectors a and b (i.e. $a^\top b$). Save this value as aDotb.

(c) Find the element-wise product of a and b $[a_1b_1, a_2b_2, a_3b_3]^T$.

(d) Find $(a^\top b)Ma$.

(e) Without using a loop, multiply each row of M element-wise by a. (Hint: The function repmat() may come in handy).

(f) Without using a loop, sort all of the values of M in increasing order and plot them.

3 Basic Image Manipulations (20 points)

(a) Read in the images, image1.jpg and image2.jpg, as color images.

(b) Convert the images to double precision and rescale them to stretch from minimum value 0 to maximum value 1.

(c) Add the images together and re-normalize them to have minimum value 0 and maximum value 1. Display this image.

(d) Create a new image such that the left half of the image is the left half of image1 and the right half of the image is the right half of image2.

(e) Using a for loop, create a new image such that every odd numbered row is the corresponding row from image1 and the every even row is the corresponding row from image2. (Hint: Remember that indices start at 1 and not 0 in MATLAB).

(f) Accomplish the same task as part e without using a for-loop (the functions reshape and repmat may be helpful here).

(g) Convert the result from part f to a grayscale image. Display the grayscale image with a title.

4 Least Squares (20 points)

(a) First, we will derive the least squares equation. Recall that in least squares, we are trying to estimate a vector x such that $y \approx Ax$ where $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, and $A \in \mathbf{R}^{m \times n}$. To do this, we define a cost function $\|y - Ax\|^2$ and try to minimize the cost function. Use matrix calculus to to derive the least squares solution of x from this cost function.

(b) In a sentence or two, why is least squares robust to noise?

Use the following script to generate sample points.

$$
\begin{aligned}
&rng(1);\\
&x = linspace(-10, 10, 120)';\\
&y = 3 * x + 4 + 10 * randn(120, 1);
\end{aligned}
$$

(c) Suppose the samples $\hat{x}, \hat{y}$ were drawn from a line $y = ax + b$ under Gaussian noise. Give a least square formulation in $||Y - XA||^2$. Explain A, X, Y in terms of $a, b, \hat{x}, \hat{y}$.

(d) Solve the above and report the estimatiated values of a and b. Finally, plot the line and all the sample points in one plot.

5 Singular Value Decomposition (20 points)

(a) Read in image1 as a grayscale image. Take the singular value decomposition of the image.

(b) Recall from the discussion section that the best rank n approximation of a matrix is $\sum_{i=1}^{i=n} u_i \sigma_i v_i^\top$, where u_i, σ_i, and v_i are the ith left singular vector, singular value, and right singular vector respectively. Save and display the best rank 1 approximation of the (grayscale) image1.

(c) Save and display the best rank 20 approximation of the (grayscale) image1.

6 Transformations (20 points)

In this problem, we will be exploring 2d transformations. Use a pentagon defined by the following points for this problem: (0,0), (1,0), (1,1), (0.5,1.5), (0,1).

(a) Plot the pentagon by plotting the points provided in order (repeat the first point at the end of the sequence to complete the loop).

(b) Define a translation matrix such that when you multiply the points on the left by your translation matrix, you translate the points 1 unit to the right and 2 units up. Plot the original points and the new translated points on the same plot in different colors. (Hint: homogeneous coordinates may be helpful).

(c) Define a rotation matrix such that when you multiply the points on the left by the rotation matrix, you rotate the original points by 15 degrees counterclockwise. Plot the original points and the new rotated points on the same plot in different colors.

(d) Define a scaling matrix such that the pentagon is scaled by a factor of 3 in the horizontal direction and 0.5 in the vertical direction. Plot the original points and the new scaled points on the same plot in different colors.

(e) Take the translation, rotation, and scaling matrices you defined above and compute the eigenvalues using Matlab. Explain why these values make sense for each matrix.