Logistics

• The midterm will be **posted** on Piazza on **2/25** at 12 PM.

• **Due** by 12 PM (noon!) on **2/27**

• Submit your midterm via **Scoryst**.

• **No public posts** on Piazza during the midterm.

• Monitor midterm post on Piazza for any clarifications or updates.

• Open book, open notes. No collaboration.
Midterm Overview

- 3 - 4 theory questions
- 1 - 2 implementation questions
- Everything up to and including PS3 is fair game.
- 6 - 10 hours estimated time.
Pinhole Camera Model

Image Credits: Hartley & Zisserman
Camera Projection

Camera Matrix

\[ P_i = MP_w \]
\[ = K_{3\times3} \begin{bmatrix} R & t \end{bmatrix}_{3\times4} P_w \]

Intrinsics

Extrinsics

\[ K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \]
Epipolar Geometry

Image Credits: Arne Nordmann
Fundamental Matrix

\[ x'^\top F x = 0 \]

Epipolar line

- Rank 2 homogeneous matrix
- 7 degrees of freedom
- Estimated using the [normalized] 8 point algorithm
Fundamental Matrix Estimation

The Eight Point Algorithm

\[ x' \top F x = 0 \]

\[
\begin{bmatrix}
  u' & v' & 1
\end{bmatrix}
\begin{bmatrix}
  F_{11} & F_{12} & F_{13} \\
  F_{21} & F_{22} & F_{23} \\
  F_{31} & F_{32} & F_{33}
\end{bmatrix}
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
  F_{11} \\
  F_{12} \\
  F_{13} \\
  F_{21} \\
  F_{22} \\
  F_{23} \\
  F_{31} \\
  F_{32} \\
  F_{33}
\end{bmatrix}
\begin{bmatrix}
  u' u' v' u' v' v' u v 1
\end{bmatrix} = 0
\]
The Eight Point Algorithm

- There are eight unknowns (9 entries, known up to scale)
- Each correspondence gives us a single constraint
- Get eight correspondences, stack them up in a matrix, solve using SVD.
- Enforce the rank 2 constraint.
- Use Hartley’s normalized version of the algorithm.
Structure from Motion

For each camera / image:

\[ i = 1, \ldots, m \]

For each 3D point:

\[ j = 1, \ldots, n \]

For known points:

\[ x_{ij} = M_i X_j \]

Solve for this.
SfM Factorization

\[ 2m \times n = 3 \times 3 \times 3 \]

- Measurements
- Motion
  - Affine Camera Matrices
- Structure
  - 3D Points
Vanishing Points + Lines

Vanishing Point

\[ v = K d \]

Intrinsics  Direction

Vanishing Line

\[ l = v_1 \times v_2 \]

Vanishing line for the plane

Plane Normal

\[ n = K^\top l \]
Hough Transform

- A **voting** scheme to **fit a parametric model** to data by selecting the model with the most votes.

- Assumes that **outliers** will not consistently vote for any particular model parameters.

- Perform voting in a discretized **parameter space**.

- Often used for finding lines and other shapes in images (but is much more general!)
Hough Transform

Circle Detection

- Assume that the radius is known.
- We can parametrize the circle by its center \((a, b)\).
- Each point votes in \((a, b)\) space.
- If the radius was unknown, we could have voted in the 3D \((r, a, b)\) space.
- Pick a bin size to discretize the \((a, b)\) space (we’ll use a resolution of 0.01).
Hough Transform

Circle Detection

After 1 iteration

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich
Hough Transform

Circle Detection

After 2 iterations

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich
Hough Transform

Circle Detection

Final Result

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich
RANSAC

- A randomized iterative method to fit a model to data based on random sampling of data.

- Assumes that outliers will not consistently vote for any particular model parameters (déjà vu?).

- Repeatedly samples a minimal set to estimate the model and keeps track of the one with the largest consensus set.
RANSAC

Circle Detection

The model

\[(x-a)^2 + (y-b)^2 = r^2\]

Center and radius unknown.

Minimal Set

3 points

Fitness Measure

\[D(x, y \mid a, b) = \left| r - \sqrt{(x-a)^2 + (y-b)^2} \right|\]

We can use this to find our consensus set.
RANSAC

Circle Detection

The consensus set and estimated circle for a random sample

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich
RANSAC

Circle Detection

The consensus set and estimated circle for a random sample

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RANSAC

Circle Detection

The final result

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich
RANSAC

Circle Detection

The final result

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.4025</td>
<td>0.40</td>
</tr>
<tr>
<td>b</td>
<td>0.5022</td>
<td>0.50</td>
</tr>
<tr>
<td>r</td>
<td>0.3010</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Inlier Count: 318
RANSAC

Pros

• Well suited for a wide range of model fitting problems.

• Easy to implement, can be easily parallelized.

• No constraints on parameter space.

Cons

• A large number of outliers can cause issues.

• Many hyper parameters to tune.

Hough Tranform

Pros

• It can handle missing / occluded data.

• Low sensitivity to noise and outliers.

• Ability to detect multiple instances of a model in a single pass

Cons

• Complexity of search time is exponential in the number of parameters.

• Quantization: it’s hard to pick a good grid size.