Midterm Review

CS231A Winter 2015

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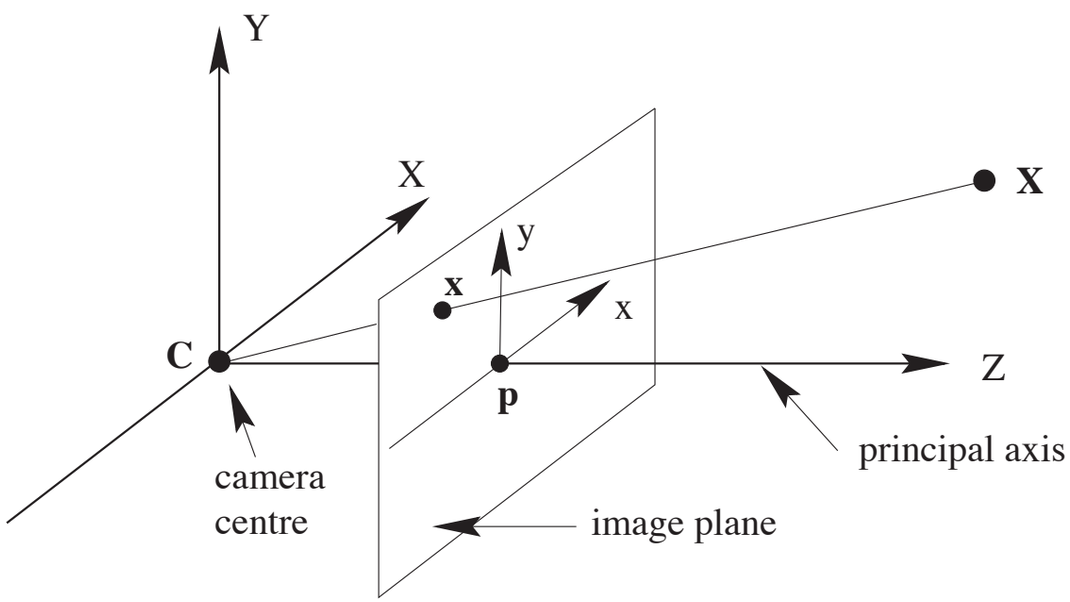
Logistics

- The midterm will be **posted** on Piazza on **2/25** at 12 PM.
- **Due** by 12 PM (noon!) on **2/27**
- Submit your midterm via **Scoryst**.
- **No public posts** on Piazza during the midterm.
- Monitor midterm post on Piazza for any clarifications or updates.
- Open book, open notes. No collaboration.

Midterm Overview

- 3 - 4 **theory** questions
- 1 - 2 **implementation** questions
- Everything up to and including **PS3** is fair game.
- **6 - 10 hours** estimated time.

Pinhole Camera Model



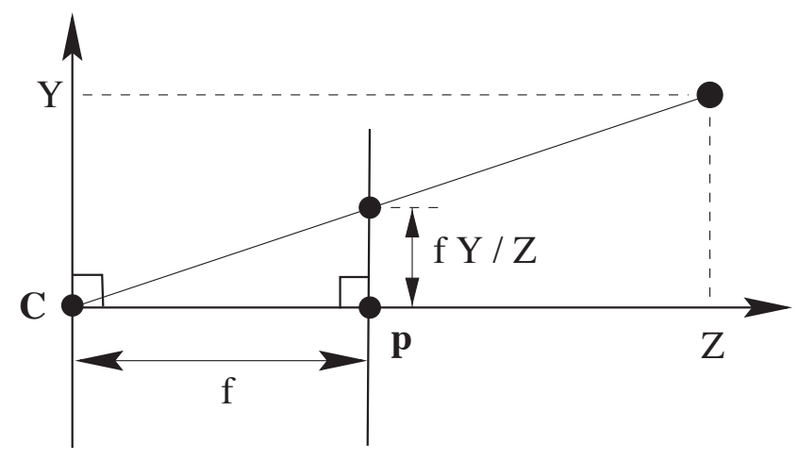


Image Credits: Hartley & Zisserman

Camera Projection

Camera Matrix

$$P_i = \mathbf{M} P_w$$

$$= \mathbf{K}_{3\times 3} \left[\mathbf{R} \mid t\right]_{3\times 4} P_w$$

Intrinsics

Extrinsics

$$\mathbf{K} = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Epipolar Geometry

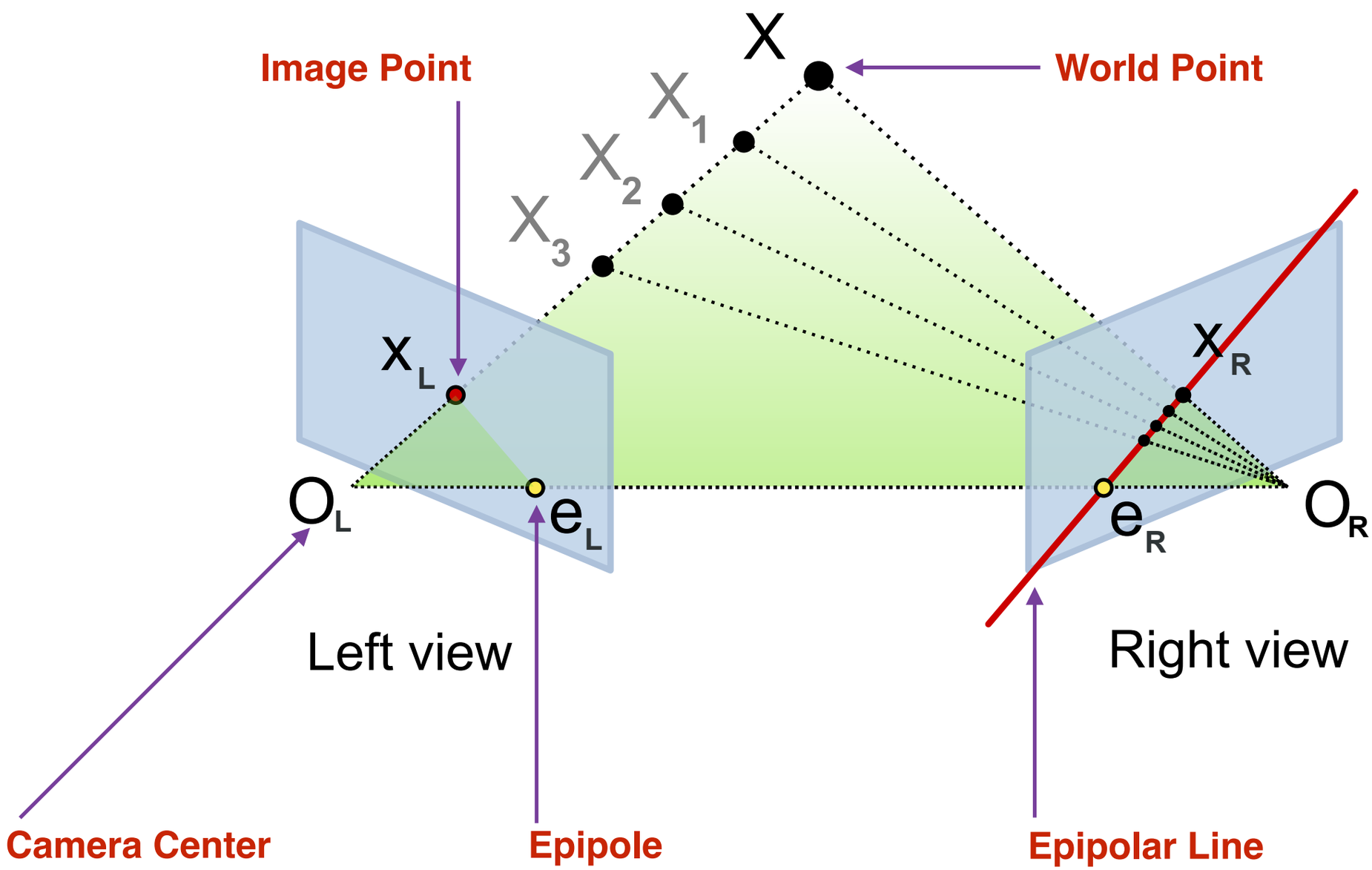


Image Credits: Arne Nordmann

Fundamental Matrix

$$x'^{\top} \underline{\mathbf{F}x} = 0$$

Epipolar line

- Rank 2 homogeneous matrix
- 7 degrees of freedom
- Estimated using the [normalized] 8 point algorithm

Fundamental Matrix Estimation

The
Eight Point
Algorithm

$$x'^\top \mathbf{F} x = 0$$

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

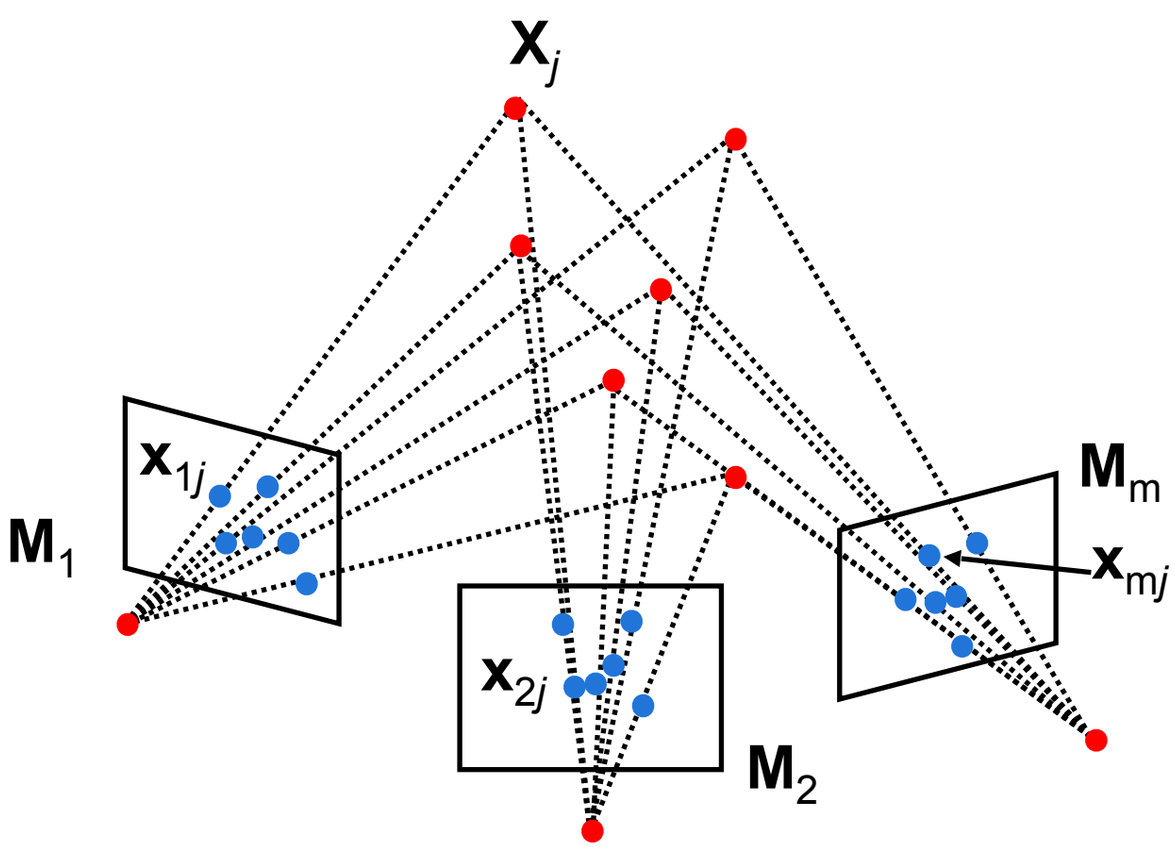
$$\begin{bmatrix} u'u & u'v & u' & v'u & v'v & v' & u & v & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Fundamental Matrix Estimation

The **Eight Point** Algorithm

- There are **eight** unknowns (9 entries, known up to scale)
- Each **correspondence** gives us a single constraint
- Get **eight** correspondences, stack them up in a matrix, solve using SVD.
- Enforce the **rank 2** constraint.
- Use Hartley's **normalized** version of the algorithm.

Structure from Motion



$$i = 1, \ldots, m$$

For each camera / image

$$j = 1, \ldots, n$$

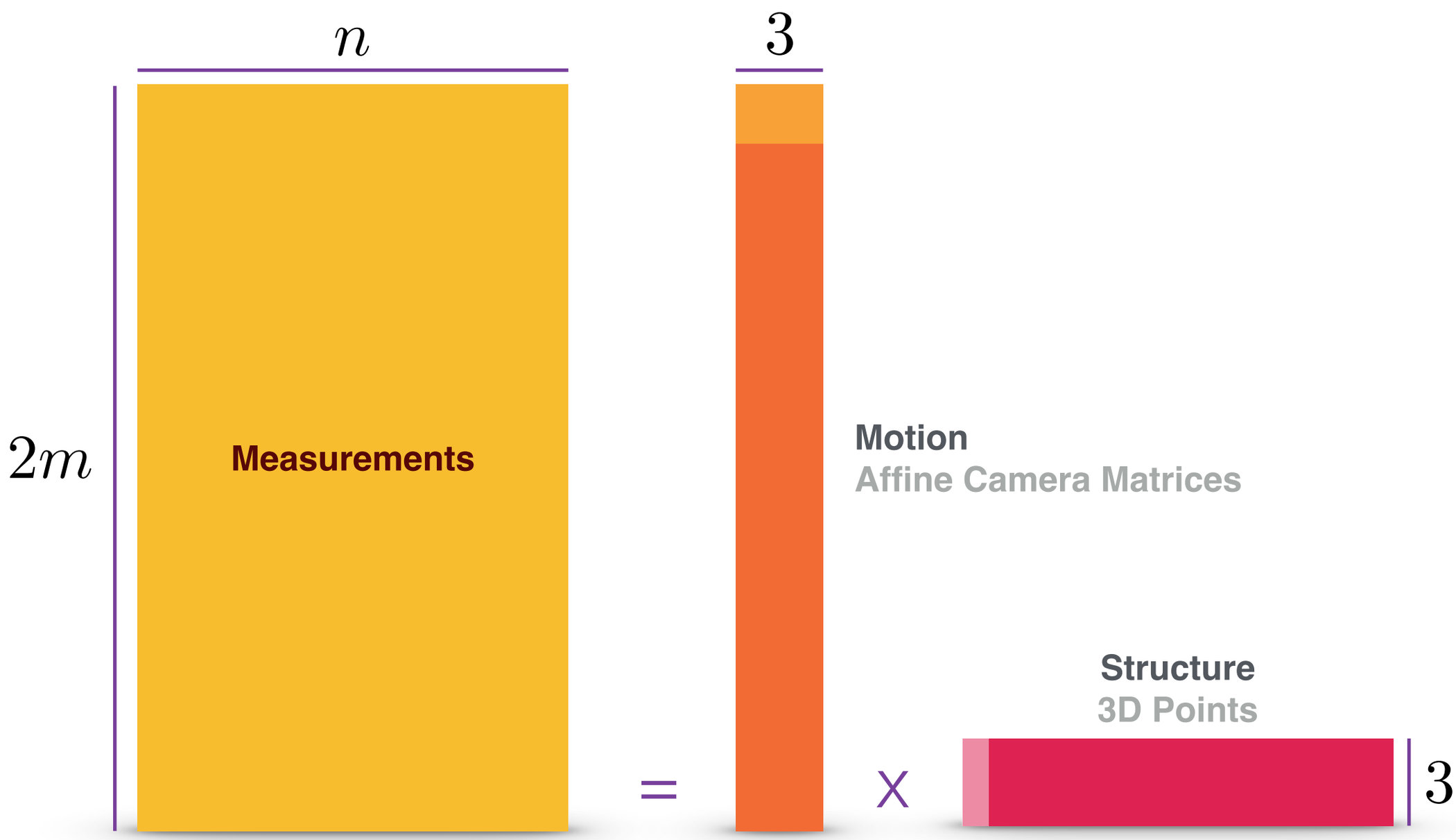
For each 3D point

$$\mathbf{x}_{ij} = \mathbf{M}_i \mathbf{X}_j$$

Known

Solve for this

SfM Factorization



Vanishing Points + Lines

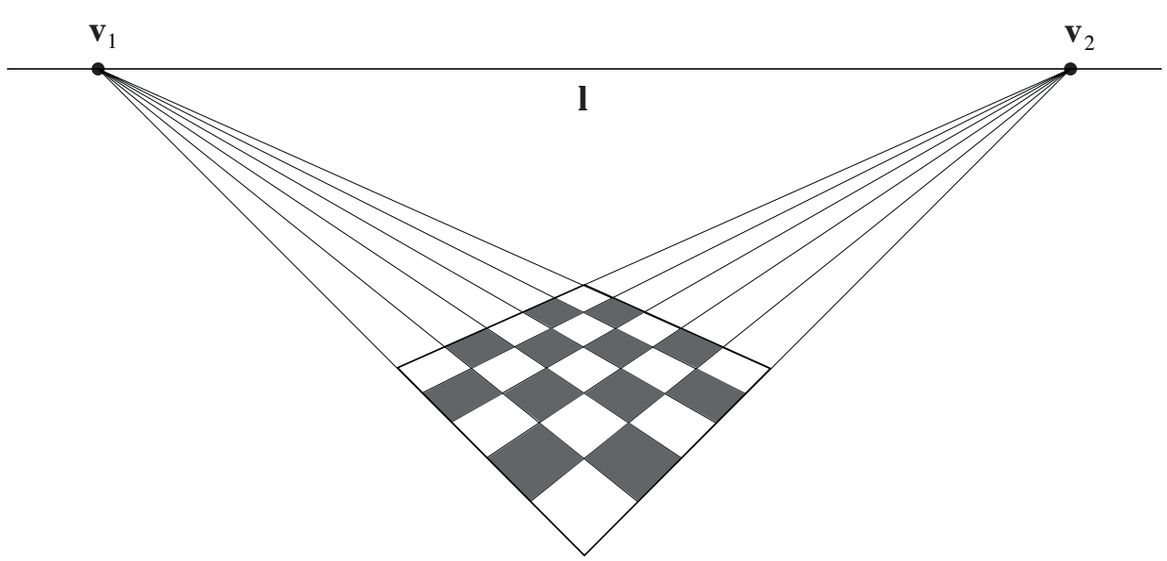


Image Credits: Hartley & Zisserman

Vanishing Point

$$v = \mathbf{K} d$$

Intrinsics Direction

Vanishing Line

$$l = v_1 \times v_2$$

Plane Normal

$$\mathbf{n} = \mathbf{K}^\top l$$

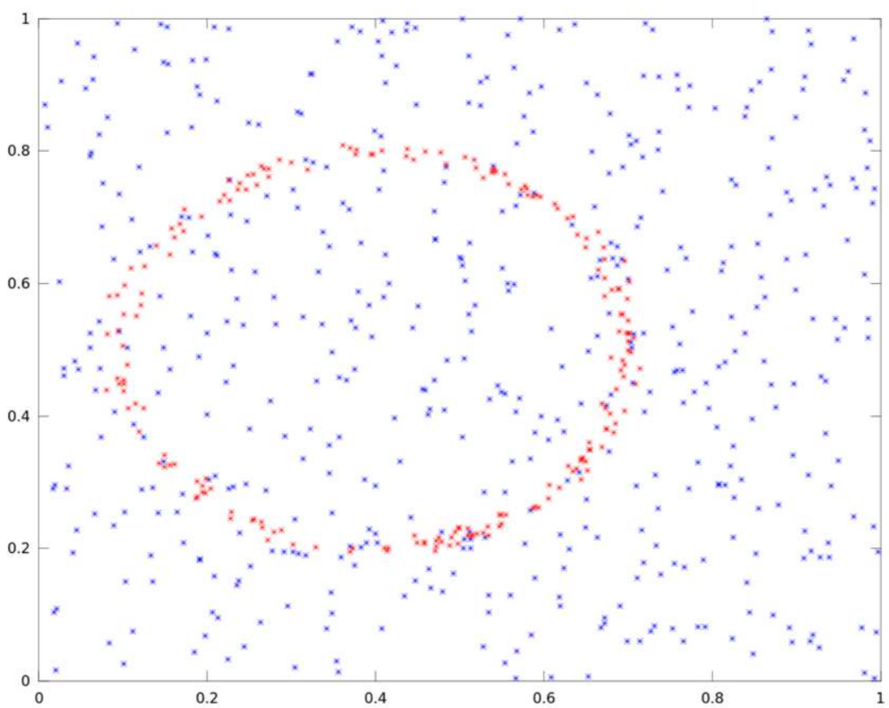
Vanishing line for the plane

Hough Transform

- A **voting** scheme to **fit a parametric model** to data by selecting the model with the most votes.
- Assumes that **outliers** will not consistently vote for any particular model parameters.
- Perform voting in a discretized **parameter space**.
- Often used for finding lines and other shapes in images (but is much more general!)

Hough Transform

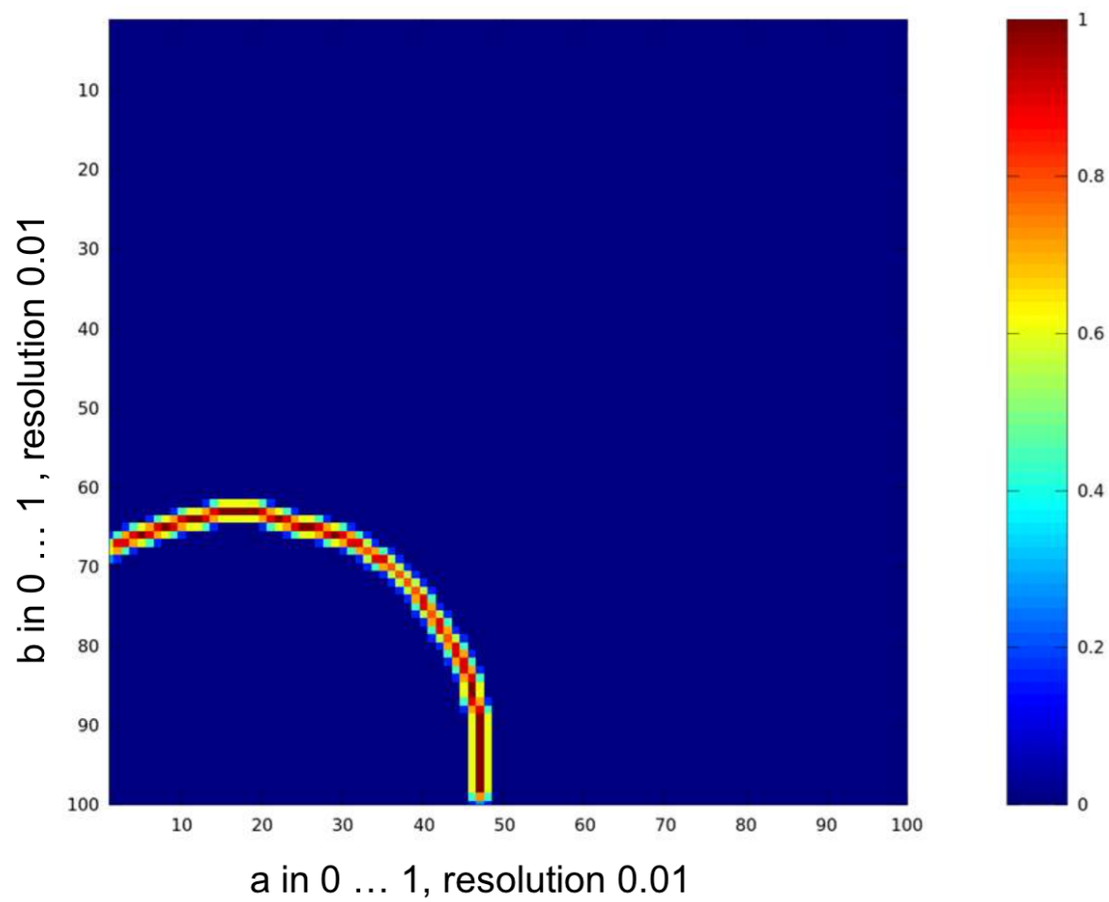
Circle Detection



- Assume that the **radius is known**.
- We can **parametrize** the circle by its **center** (a, b).
- Each point votes in (a, b) space.
- If the radius was unknown, we could have voted in the 3D (r, a, b) space.
- Pick a bin size to discretize the (a, b) space (we'll use a resolution of 0.01).

Hough Transform

Circle Detection

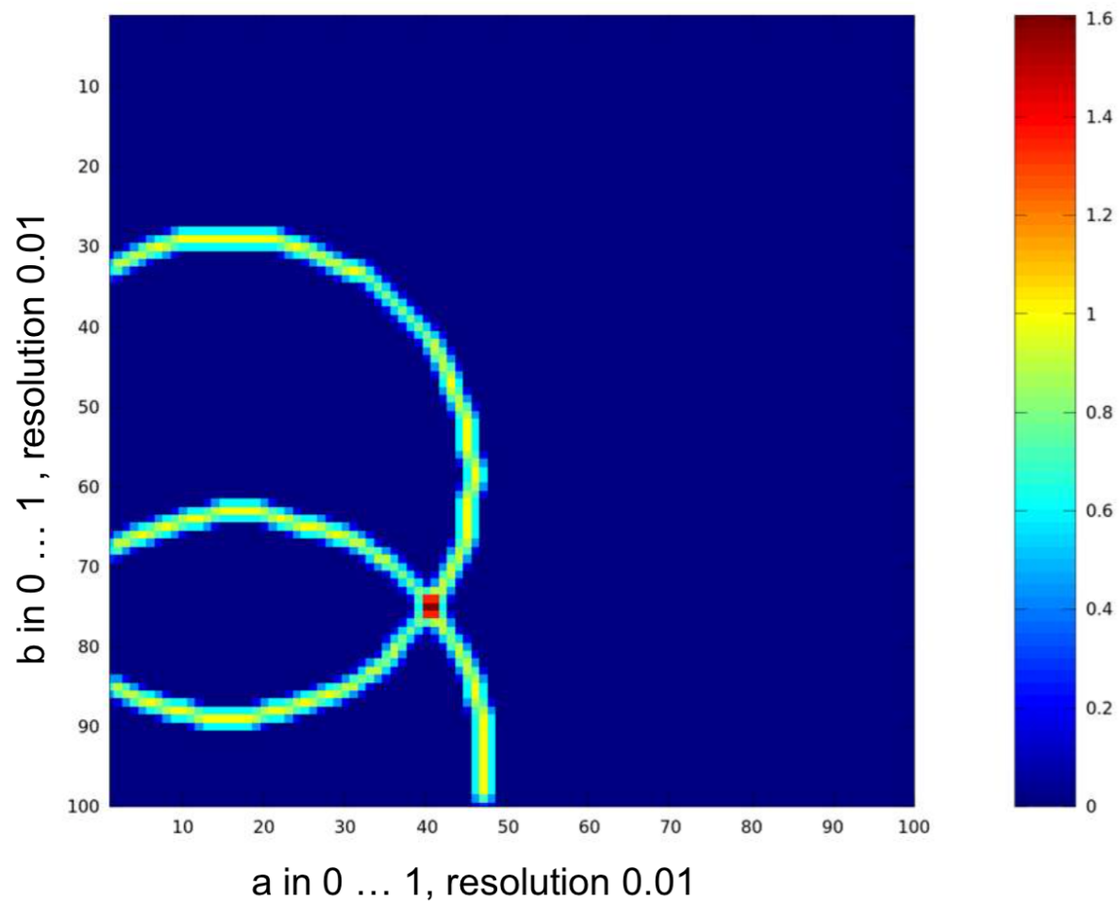


After 1 iteration

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich

Hough Transform

Circle Detection

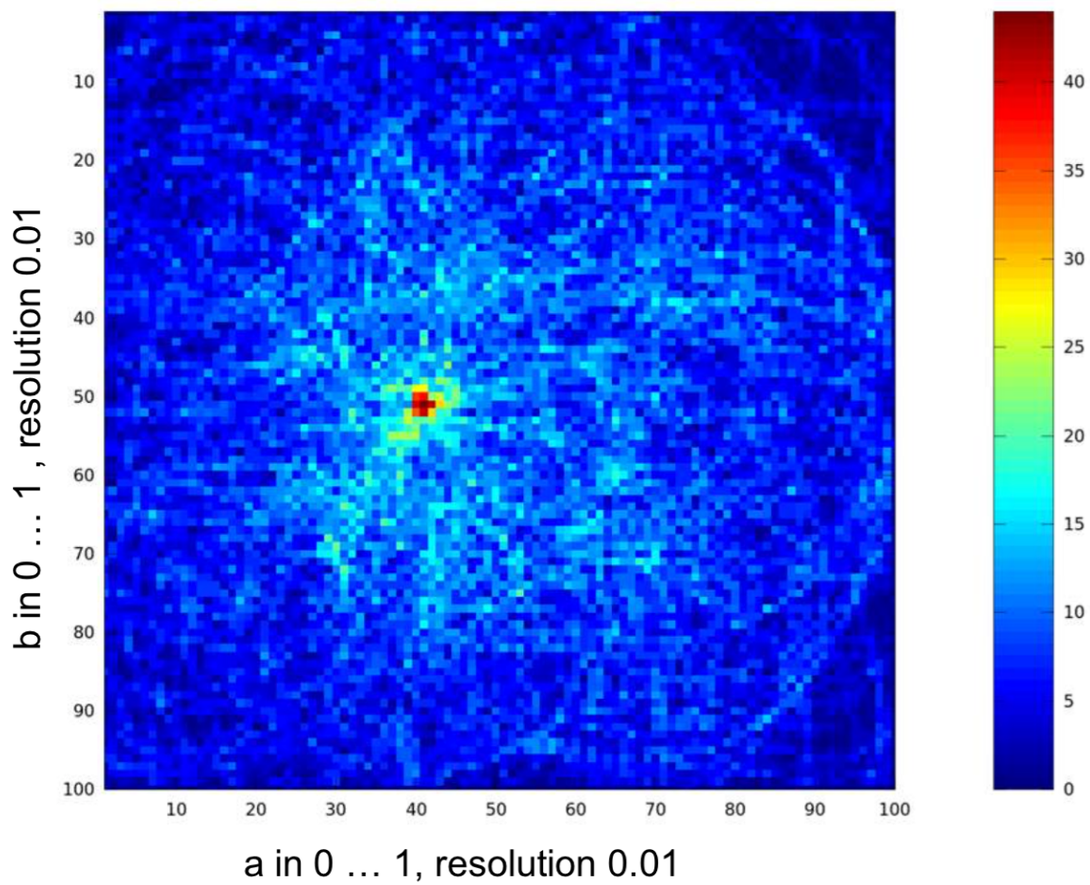


After 2 iterations

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich

Hough Transform

Circle Detection



Final Result

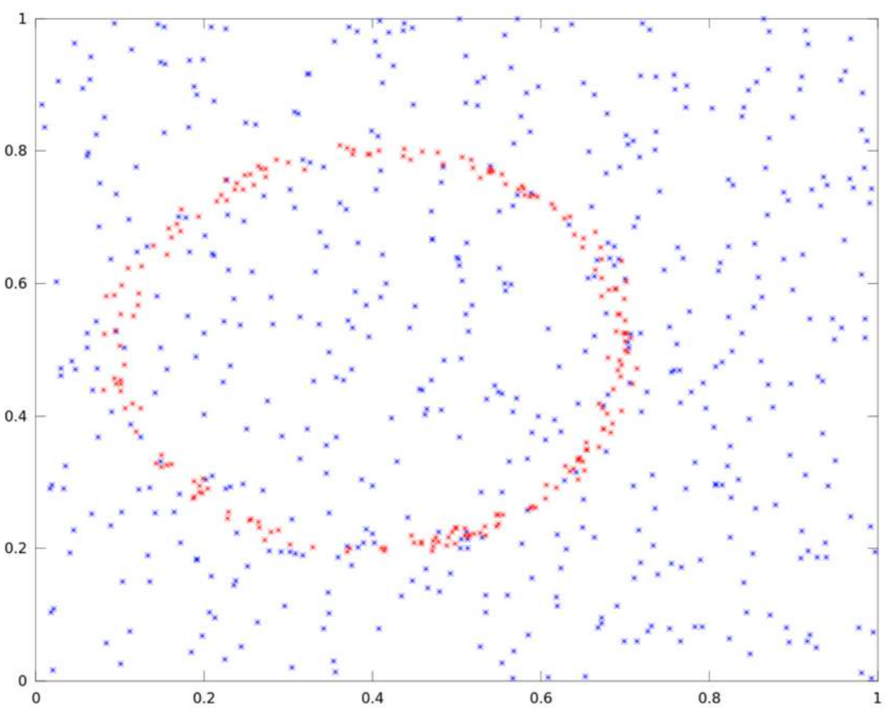
Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich

RANSAC

- A randomized iterative method to **fit a model** to data based on **random sampling** of data.
- Assumes that **outliers** will not consistently vote for any particular model parameters (déjà vu?).
- Repeatedly samples a **minimal set** to estimate the model and keeps track of the one with the largest **consensus set**.

RANSAC

Circle Detection



The model

$$(x-a)^2 + (y-b)^2 = r^2$$

Center and radius unknown.

Minimal Set

3 points

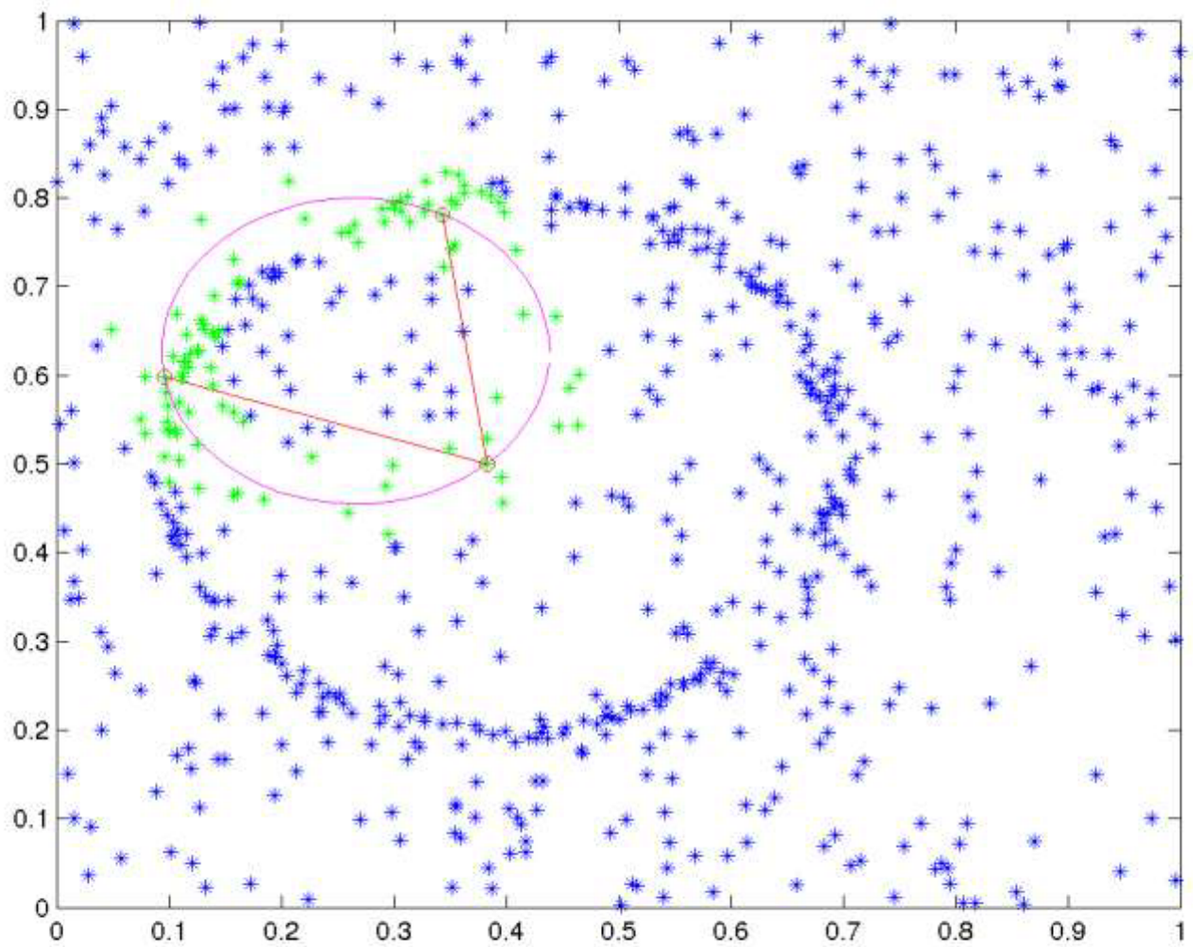
Fitness Measure

$$D(x, y \mid a, b) = \left| r - \sqrt{(x-a)^2 + (y-b)^2} \right|$$

We can use this to find our consensus set.

RANSAC

Circle Detection

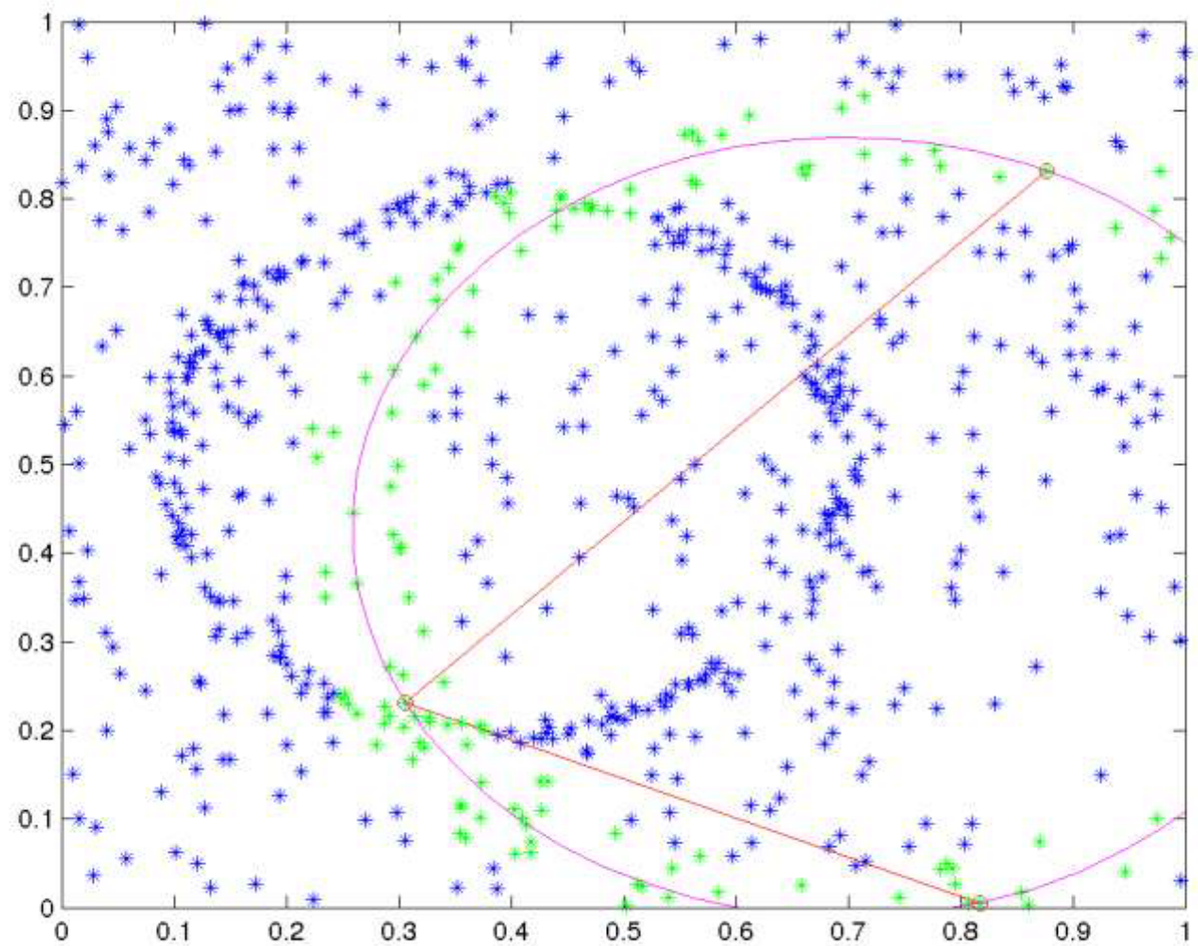


The consensus set and estimated circle for a random sample

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich

RANSAC

Circle Detection



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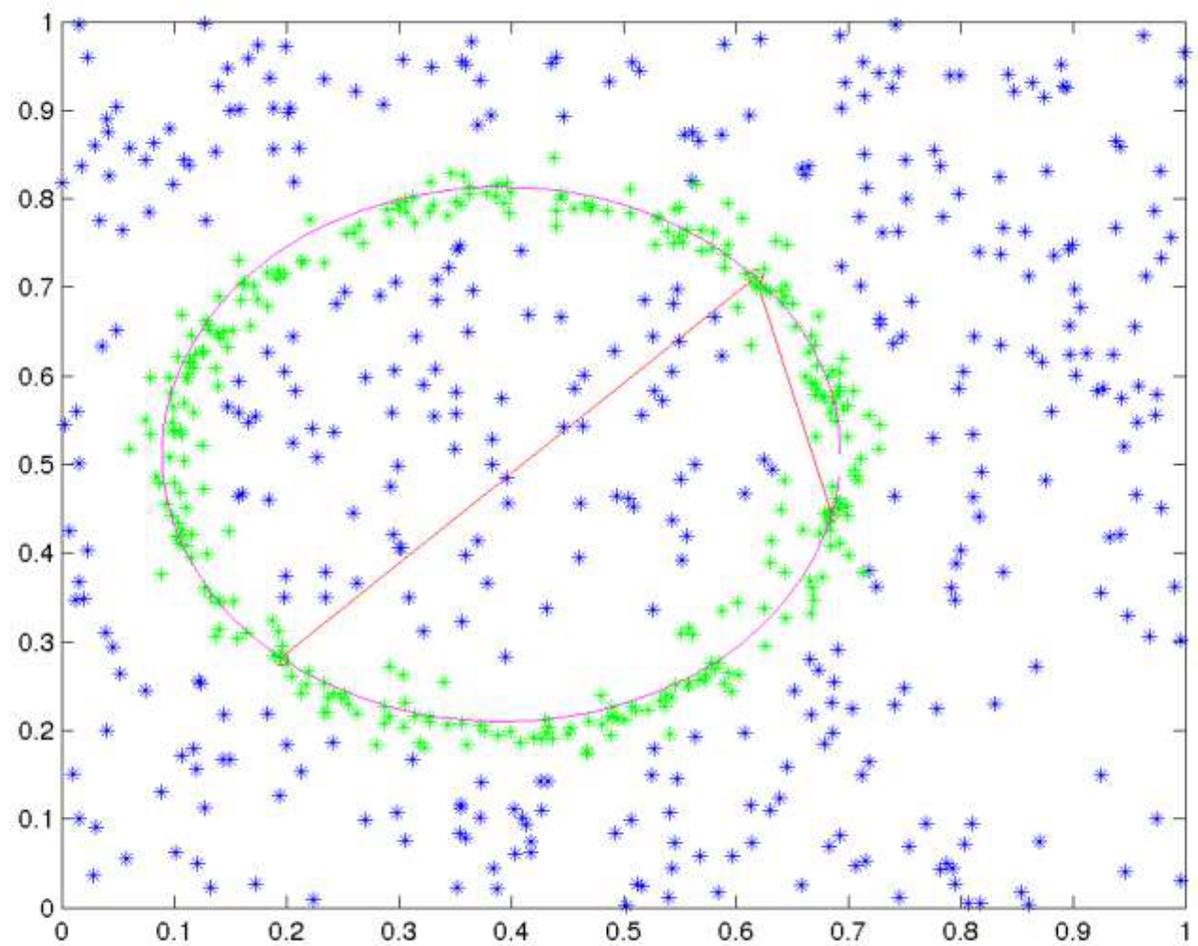
RANSAC

Circle Detection



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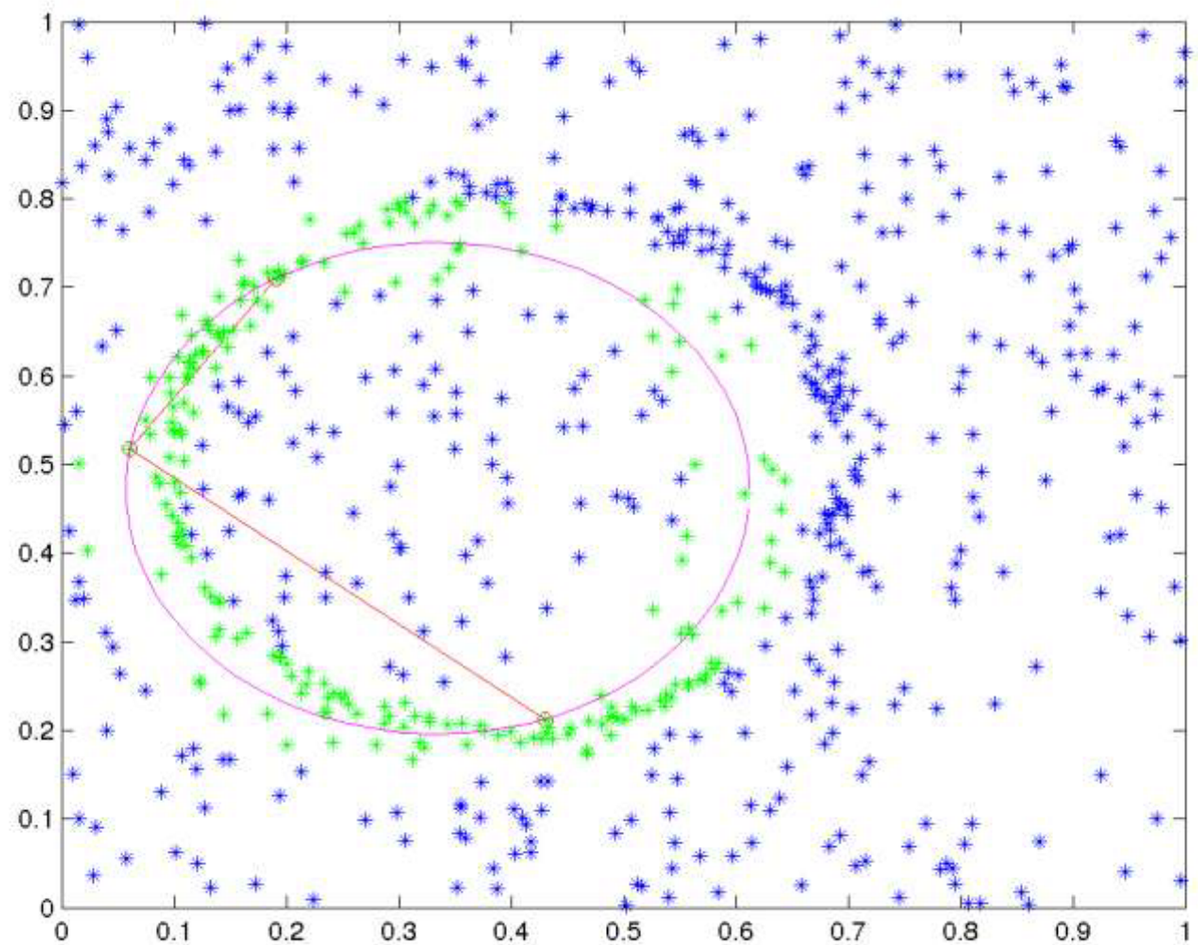
RANSAC

Circle Detection



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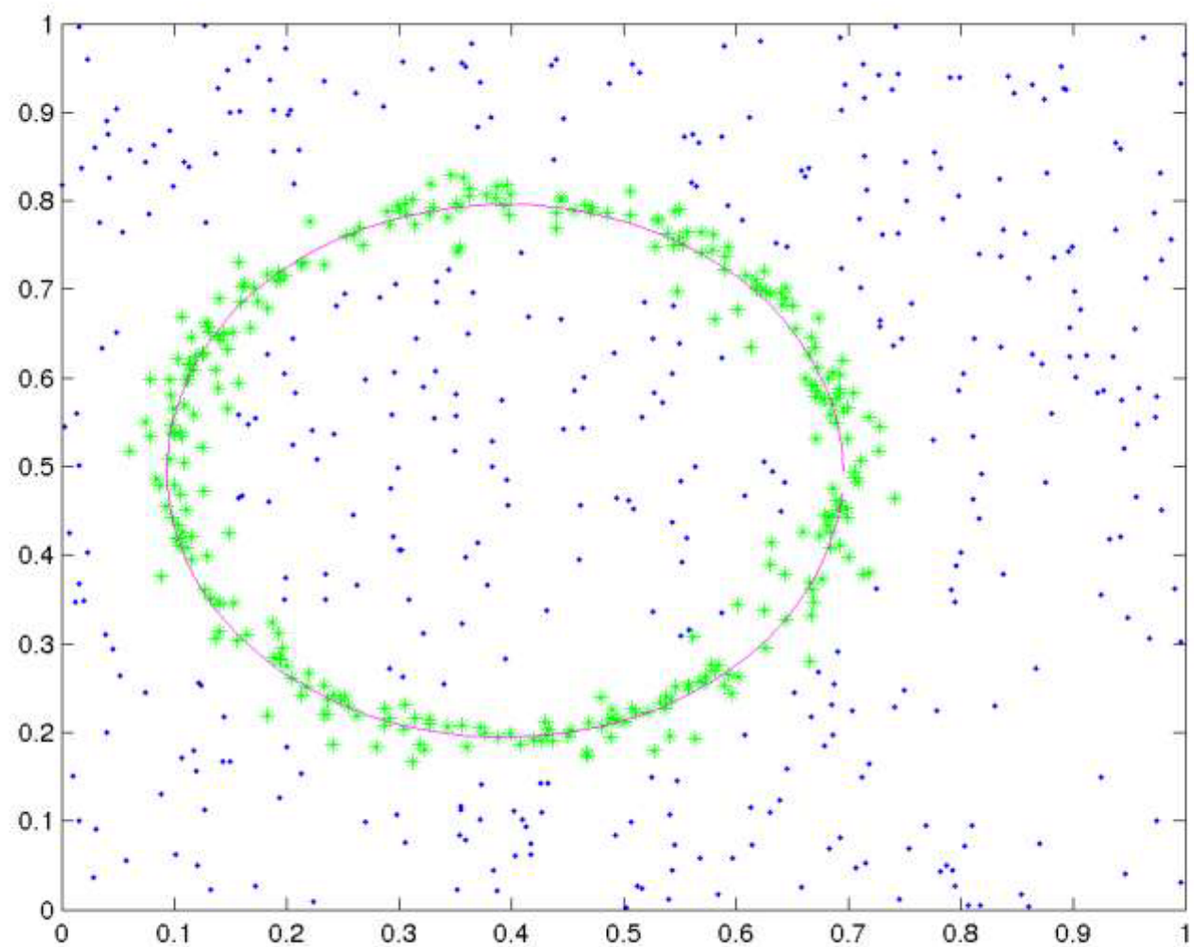
RANSAC

Circle Detection

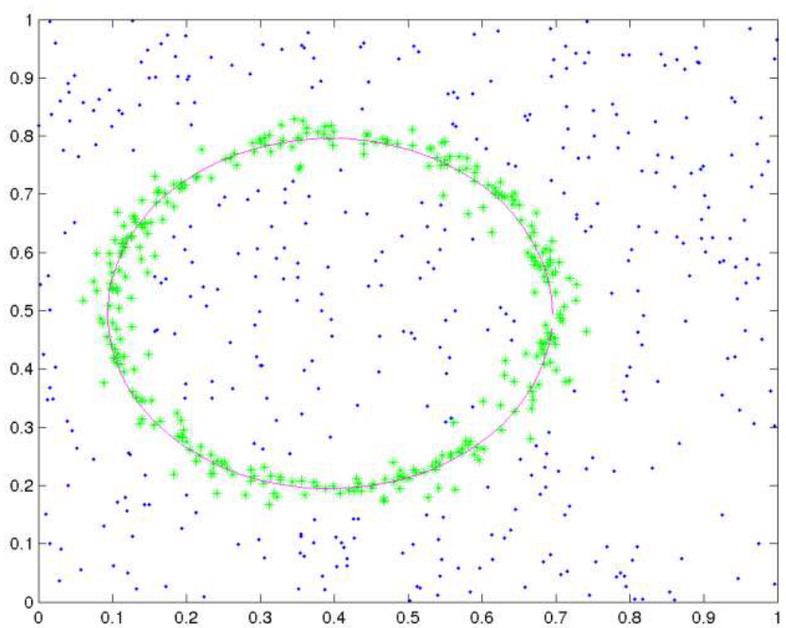


The final result

Image Credits: Dr. Cédric Pradalier, Autonomous Systems Lab, ETH Zürich

RANSAC

Circle Detection



The final result

Parameter	Estimated	Actual
a	0.4025	0.40
b	0.5022	0.50
r	0.3010	0.30

Inlier Count: 318

RANSAC

Pros

- Well suited for a wide range of model fitting problems.
- Easy to implement, can be easily parallelized.
- No constraints on parameter space.

Cons

- A large number of outliers can cause issues.
- Many hyper parameters to tune.

Hough Tranform

Pros

- It can handle missing / occluded data.
- Low sensitivity to noise and outliers.
- Ability to detect multiple instances of a model in a single pass

Cons

- Complexity of search time is exponential in the number of parameters.
- Quantization: it's hard to pick a good grid size.